THE COSMIC GRAVITY–VELOCITY RELATION

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ABSTRACT

We present a numerical study of the relation between the cosmic peculiar velocity field and the gravitational acceleration field. We show that on mildly non-linear scales (3–10 Mpc Gaussian smoothing), the distribution of the Cartesian coordinates of each of these fields is very well approximated by a Gaussian. In particular, their negentropies are small compared to those of the velocity divergence and density fields. We find that at these scales the relation between the velocity and acceleration fields follows linear theory to high accuracy. The non-linear correction of Kudlicki et al. (2000a) works still better: its reconstruction errors are several times smaller than those of the linear theory approximation. © 2002 Published by Elsevier Science Ltd on behalf of COSPAR.

INTRODUCTION

On very large, linear scales, the relation between the density contrast δ and the peculiar velocity $\mathbf{v}$ in co-moving coordinates can be expressed in differential form,

$$\delta(\mathbf{r}) = -(\dot{H}_0 f)^{-1} \nabla \cdot \mathbf{v}(\mathbf{r}), \quad (1)$$

or in integral form,

$$\mathbf{v}(\mathbf{r}) = \mathbf{g}(\mathbf{r}) \equiv \frac{H_0 f}{4\pi} \int d^3 \mathbf{r}' \frac{\delta(\mathbf{r}') (\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}. \quad (2)$$

Here, $\mathbf{g}$ is the gravitational field, rescaled so that $\delta = -(\dot{H}_0 f)^{-1} \nabla \cdot \mathbf{g}(\mathbf{r})$. The coupling constant, f , carries information about the underlying cosmological model, and is related to the Cosmological Density Parameter and Cosmological Constant by $f(\Omega, \Lambda) \simeq \Omega^{0.6} + \frac{\Lambda}{70} \left(1 + \frac{\Omega}{2}\right)$. Hence, comparing the observed density and velocity fields of galaxies allows one to constrain Ω , or the degenerate combination $\beta \equiv \Omega^{0.6}/b$ in the presence of galaxy biasing (see Strauss & Willick 1995 for a review). This is done by extracting the density field from full-sky redshift surveys, and comparing it to the observed velocity field. Present-day peculiar velocity catalogs allow us to compare the fields smoothed on scales of about 3 to 10 Mpc: any smaller would be too strongly affected by nonlinear effects, while larger smoothing would reduce the number of independent volumes over which the comparison could be made. It is then important to find the extensions of equations (1) and (2) for these, mildly nonlinear, scales.

Kudlicki et al. (2000a, hence KCPR) have shown that the relation between $\theta \equiv -(\dot{H}_0 f)^{-1} \nabla \cdot \mathbf{v}(\mathbf{r})$, (see equation 1) and $\delta = -(\dot{H}_0 f)^{-1} \nabla \cdot \mathbf{g}(\mathbf{r})$ is nonlinear on small scales, and have proposed a semi-empirical formula accurately describing the dependence of $\theta_\delta \equiv \langle \theta | \delta \rangle$ on δ :

$$\theta_\delta = \alpha \left[(1 + \delta)^{1/\alpha} - 1 \right] + \epsilon, \quad (3)$$

where the constant ϵ is approximately $\epsilon = \frac{\alpha-1}{2\alpha} \sigma_\delta^2$, and $\sigma_\delta^2 \equiv \langle \delta^2 \rangle$ denotes the variance of the density field. Simulations have shown that $\alpha = 1.9$ is a good fit over a large range of smoothing scales. Because $\mathbf{v}$ is irrotational on mildly non-linear scales,

$$\mathbf{v}(\mathbf{r}) = \frac{H_0 f}{4\pi} \int d^3 \mathbf{r}' \frac{\theta_\delta(\mathbf{r}') (\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}, \quad (4)$$

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Fig. 1. The probability distribution of one component of the 5 Mpc Gaussian smoothed velocity field (left panel) and gravity field (right panel) from six simulations and their deviations from Gaussianity. The Gaussians plotted here, $G(v/\sigma_v)$ and $G(g/\sigma_g)$, are properly normalized Gaussians of the same variance.

where equation (3) gives an expression for θ_δ .

The amplitude of the velocity field smoothed on a given scale R depends on the density field power spectrum, $P(k)$, as $\langle v^2 \rangle \propto \int dk P(k) \tilde{W}^2(kR)$, while for the density field the relation is as follows: $\langle \delta^2 \rangle \propto \int dk k^2 P(k) \tilde{W}^2(kR)$. This means that the velocity field is more heavily weighted by modes with low values of the wavenumber k , i.e., large scales which are fully in the linear regime. Therefore, we expect the relation between $\mathbf{v}$ and $\mathbf{g}$ to be closer to linear than that between the density and velocity divergence, but the magnitude of its nonlinearity needs to be examined.

In this paper, we investigate the nonlinearities of the relationship between the velocity and gravity fields, $\mathbf{v}$ and $\mathbf{g}$, using a set of numerical simulations. We first ask how well the probability distribution functions (PDFs) of the Cartesian components of $\mathbf{v}$ and $\mathbf{g}$ are fit with a Gaussian. Next, we directly measure the relation between $\mathbf{v}$ and $\mathbf{g}$ on various scales, and test the extent to which linear theory, or non-linear extensions to it, may hold. This is important in determining whether the existing velocity-velocity comparisons which use linear theory give biased results. The results of this paper are based on a paper (Kudlicki et al. 2000c) submitted to MNRAS.

We investigate the gravity-velocity relation using simulations done with the Eulerian CPPA code (Kudlicki et al. 1996, Kudlicki et al. 2000a,b). We performed six simulations using a $(200 h^{-1} \text{Mpc})^3$ box and the APM power spectrum (Baugh and Gaztañaga 1996) as the initial conditions.

MARGINAL DISTRIBUTIONS OF $\mathbf{V}$ AND $\mathbf{G}$

The typical correlation length of the density field is of the order of 5 Mpc, while $\mathbf{g}$ is influenced by density fluctuations in a much larger region. Thus, $\mathbf{g}$, and similarly $\mathbf{v}$, come from integration over an effective domain containing a large number of essentially independent regions. Hence, the Central Limit Theorem suggests that they should have close to Gaussian distributions.

We test the Gaussianity hypothesis with our simulations. We plot the measured distribution functions for individual Cartesian components of the peculiar velocity and gravity fields, which we label v and g . On mildly nonlinear scales, the distributions are well-fit by Gaussians. Figure 1 shows the distributions for 5 Mpc Gaussian smoothing. The agreement with a Gaussian distribution is remarkable.

There are several ways to quantify the closeness of the distribution to a Gaussian. We borrow the concept of *negentropy* from information theory for such a measure. Let us define the entropy $H[f]$ of a probability

Fig. 2. Negentropies of the cosmic fields. Upper panel compares the negentropies of gravity g and density contrast δ ; lower - negentropies of velocity v and of its divergence θ .

distribution f :

$$H[f] = - \int f(x) \log f(x) dx. \quad (5)$$

It can be shown that the entropy is maximal for Gaussian fields. Hence the difference between the entropy of a given field and the entropy of a Gaussian of the same variance, the *negentropy* $J[f]$, can be used as a measure of departure from Gaussianity:

$$J[f] = H[Gauss] - H[f]. \quad (6)$$

We plot measured negentropies of the cosmic fields as functions of Gaussian smoothing scale in figure 2. The negentropy of the density field is compared to the negentropy of the gravity in the upper panel. The values for velocity and its divergence are compared in the lower panel of this figure. We conclude that on mildly non-linear scales the non-Gaussianity of v and g is completely negligible compared to the non-Gaussianity of θ and δ respectively.

THE ONE-COMPONENT V-G RELATION

In the linear regime $\mathbf{v} = \mathbf{g}$ and thus each of the Cartesian components of these quantities, which we denote by v and g , respectively, are also equal. The relation between δ and θ becomes non-linear on scales of several Mpc. Nevertheless, the probability distributions of the Cartesian components of both $\mathbf{v}$ and $\mathbf{g}$ are still nearly Gaussian, and their joint probability distribution is close to a bivariate Gaussian as well, as shown in Figure 3. In the strictly bivariate Gaussian case the mean relationship between the variables is linear; we shall check this for v and g measured in our simulations and estimate the coefficient of the relation. The relationship between these two quantities has some finite scatter (Fig. 3), thus we will characterize the mean v at given g , $\langle v|g \rangle$. From symmetry, it must be an odd function.

Hence, we shall adopt third-order polynomial

$$\langle v|g \rangle = d_1 g + d_3 g^3 \quad (7)$$

as the simplest odd non-linear model, and use the parameter d_3 as a measure of the non-linearity of the relation.

Fig. 3. The joint probability distribution of a Cartesian component of the cosmic gravity and velocity fields, both smoothed with a $5h^{-1}\text{Mpc}$ Gaussian filter. Heavy contours represent combined numerical results from six simulations, thin ones – the 2-D Gaussian of the same correlation coefficient. Contours are plotted at probability levels of 0.2, 0.4 and 0.6.

Fig. 4. The parameters d_1 and d_3 of the polynomial approximation to the mean one-component $g - v$ relation (Eq. 7) calculated from the simulations.

Fig. 5. One Cartesian component of the velocities reconstructed from the density field, combined data from 6 simulations, observed with a $4 h^{-1} \text{Mpc}$ Gaussian filter. Top: mean true velocity given predicted velocity. Bottom: Errors of the reconstruction. All velocities are in kilometers per second. The rms true velocity for this smoothing equals 170 km/s . Thick solid line: identity, open circles: linear theory model, dashed curve in lower panel: polynomial fit to this model (Eq. 7). The error of the polynomial model is measured by the difference between the dashed curve and the open circles. Triangles represent reconstruction from gravity field computed using the non-linear formula of KCPR (Eq. 3 in the present paper), with $\alpha = 1.9$.

Since, as we have already stated, the joint probability distribution of v and g is close to Gaussian, we expect d_3 to be small (i.e., giving a nearly linear relation). The value of d_1 predicted by linear theory is $d_1 = 1$, but we are dealing here with fields well in the mildly non-linear regime, and this need not hold.

The measured parameters d_1 and d_3 are shown in Figure 4, as functions of the smoothing scale, R . As expected, d_3 is much smaller than unity. Moreover, $d_1 \simeq 1$ even on small scales, which implies that the systematic bias in estimating β based on the linear theory approximation (2) is small.

We plot the mean true velocity as a function of the velocity predicted with the linear model (i.e., the gravity) for a 4 Mpc Gaussian filter as open circles in Figure 5. Combined data from 6 realizations of our model are binned with respect to predicted velocity and averaged in the bins. Linear theory, $\mathbf{v} = \mathbf{g}$, is a good fit for velocities up to at least $2\sigma_v$. The long-dashed curve in the lower panel shows how accurately the relation can be modeled by the polynomial formula (7).

For comparison, the triangles are the velocities predicted using the fully non-linear formula (inserting equation 3 in 4), using $\alpha = 1.9$; this gives still a better fit. Thus we have demonstrated that the non-linear effects in the $v - g$ relation can be corrected for either at the level of the $\delta - \theta$ relation (as proposed by KCPR), or by transforming the gravity field according to equation (7). However, the value of $\alpha = 1.9$ is universal for the former method, while for the latter, one needs to supply a model for the dependence of d_1 and d_3 on σ .

To quantify the error in estimation of β made by applying the linear approximation (2), we compute a volume-weighted linear fit to the scaled cosmic velocity as function of gravity. On the 4 Mpc scale, its slope is 0.91, which corresponds to a systematic 9% error in β . For comparison, we calculate the same, using velocities predicted by equation (3) instead of gravities, and obtain an error smaller than 3% on the 4 Mpc scale. In principle, we could refine equation (3) further to make the agreement with the simulations essentially perfect, but such a formula would be more complex, and would unlikely hold for a range of smoothing scales and power spectra.

SUMMARY AND CONCLUSIONS

We have demonstrated that, on scales larger than 4 Mpc, the non-Gaussianities of the cosmic velocity and gravity fields are small compared to the non-Gaussianities of velocity divergence and gravity. Guided by this result, we have shown that the relation between v and g is nearly linear. Moreover, its proportionality coefficient is close to that predicted by linear theory. Specifically, we have shown that the systematic errors in velocity-velocity comparisons due to assuming the linear model do not exceed 10% in β . Reconstruction of the velocity using the non-linear formula of KCPR gives still better results (3% accuracy, with deviations of well under 100 km/s at 4 Mpc Gaussian smoothing even at the highest peculiar velocities).

ACKNOWLEDGMENTS

We thank Tomek Plewa for his contribution to the numerical code we use and Michał Różyczka for fruitful and animating discussions. This work was supported by KBN grant 2P-03D-014-19, and NSF grant AST96-16901. The simulations were performed at the *Interdisciplinary Centre for Mathematical and Computational Modelling*, Pawińskiego 5A, PL-02-106, Warsaw.

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